PHYS302 Fall 2023

<u>Homework 10</u>

1. You are measuring the interference (the "beat note") of two EM waves with intensities I_1 and I_2 on a photodetector. Think about how to maximize the signal to noise. For this question, the signal will be defined as the interference fringes (the interference term), and the noise will be the average intensity without interference (the sum of I_1 and I_2).

    a. Find an equation that gives you the ratio of signal to noise of the measurement as you hold I_1 constant and vary I_2 as a fraction of I_1.

    b. Plot the signal to noise of the measurement as you hold I_1 constant and vary I_2 as a fraction of I_1.

    (Note: this problem was chosen with specific laboratory applications in mind. It shows why you sometimes want to reduce the total power in one arm of an interferometer, even though that will decrease the absolute power of the signal, in order to increase your signal to noise.)

2. Interference from slits:
    a. 543 nm monochromatic light illuminates two (infinitely narrow) vertical slits that are separated by 0.500 mm and forms an interference pattern on a screen 1.00 m away. Describe the functional form of the interference pattern. What is the spacing between the zeroth and first order bright fringes? What is the spacing between the first and second order bright fringes?

    b. 543 nm monochromatic light illuminates a single vertical slit that is 0.050 mm wide and forms an interference pattern on a screen 1.00 m away. Describe the functional form of the interference pattern. What is the spacing between the zeroth and first order bright fringes? What is the spacing between the first and second order bright fringes?

    c. Now combine the two: You illuminate 2 vertical slits, each 0.050 mm wide and separated (from their centers) by 0.500 mm, with 543 nm light. The interference pattern is projected on a screen 1.00 m away.

        Describe the interference pattern. Draw a plot of the intensity of this interference pattern as a function of the horizontal distance from the optical axis (the line that is halfway between the two slits). (The intensity you plot does not need to be exact, but give a good idea of the relative intensity of the fringes to each other.)

Thin films and anti-reflective coatings:

3. A soap film surrounded by air has an index of refraction of 1.34. If a region of the film appears bright red (λ = 633 nm) in normally reflected light, what is its minimum thickness there?

4. Show that a film of thickness $\lambda/4$ and index n_f will always reduce the reflectance of the substrate on which it is deposited, as long as $n_0 < n_f < n_s$. (λ is the wavelength of light in the film.) Consider the simplest case of normal incidence and $n_0 = 1$. Show that this is equivalent to saying that the waves reflected back from the two interfaces cancel one another.

5. Determine the refractive index and thickness of a film to be deposited on a glass surface (n_g = 1.54) such that no normally incident light of wavelength 540 nm is reflected.

The Fabry-Perot Interferometer:

6. Given that the mirrors of a Fabry–Perot Interferometer have an amplitude reflection coefficient of r = 0.8944, find:

    a. the coefficient of finesse

    b. the finesse

    c. the full width at half maximum (FWHM)

    d. the free spectral range (FSR)

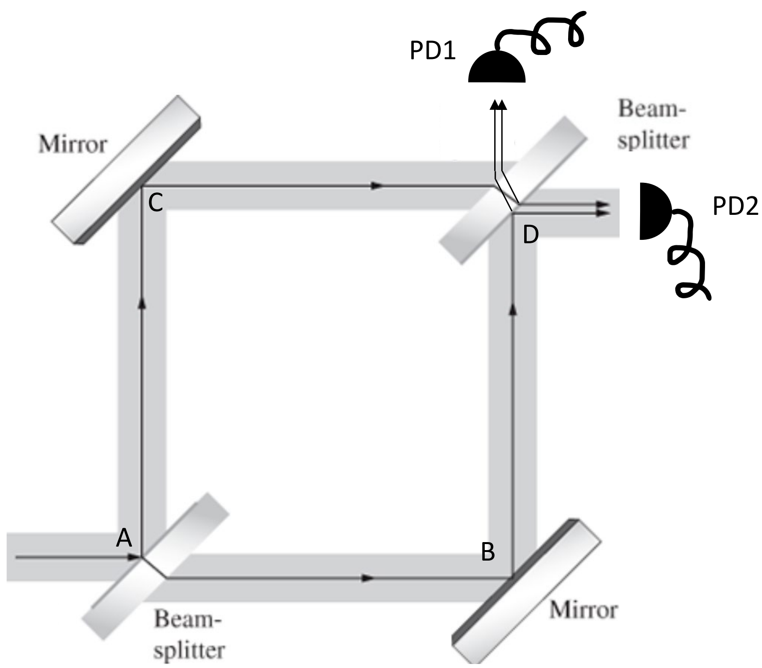
    e. the contrast factor, C, defined by:

$$C \equiv \frac{(I_t/I_i)_{max}}{(I_t/I_i)_{min}}$$

Added later: **For this problem, you can assume that the interferometer is 10 cm long and the light inside it has a wavelength of 550 nm (frequency of 545 THz).**
This will allow you to give the FSR and FWHM in wavelength or frequency, instead of radians.

7. Interferometers and conservation of energy:

Below is a Mach-Zehnder Interferometer (MZI), which is a common interferometer setup used in optics labs. The same laser is split by a 50/50 beamsplitter (non-polarizing), and half the intensity is sent through leg 1 (ABD) while the other half is sent around leg 2 (ACD). The two beams are recombined on another 50/50 beamsplitter.



Suppose the optical path length (not including phase shifts due to reflection) is exactly equal for legs 1 and 2. Also suppose that the intensity of the laser before the MZI is I_0, and the light is in a very narrow beam.

<u>What intensity does detector 2 see? What intensity does detector 1 see?</u>

(Hint: this will all come down to the phase shifts due to reflection. You must take into account that the beamsplitters will either reflect light on their front surface or their back surface, but in order for the two paths to interfere after the final beamsplitter, they must reflect off the same surface of that splitter.)

Note how the answer is consistent with energy conservation.

Extra credit 1: More fun with the Mach-Zehnder Interferometer

(An extension of problem 7)

If we expand the beam cross section and use the interferometer to make an image (see figure below), we will get two interference patterns at the two ports of the final beamsplitter.

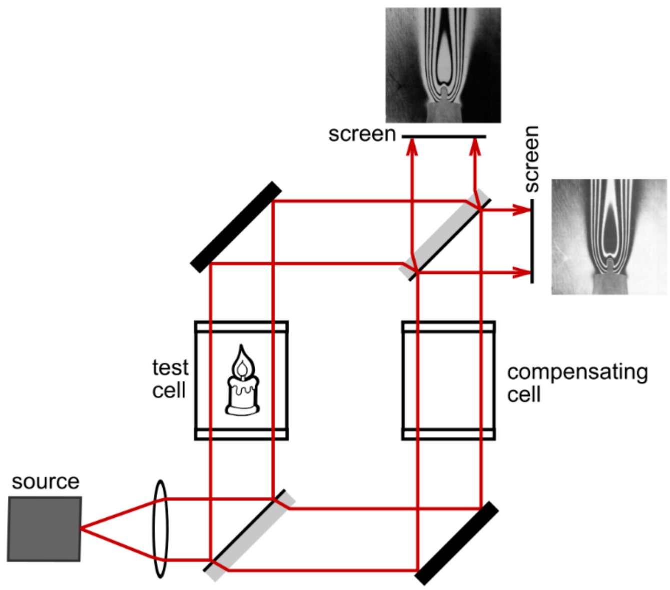


Figure caption (from Wikipedia): The Mach–Zehnder interferometer is frequently used in the fields of aerodynamics, plasma physics and heat transfer to measure pressure, density, and temperature changes in gases. In this figure, we imagine analyzing a candle flame. Either output image may be monitored. Credit: Wikipedia, Stigmatella aurantiaca with modification by Kid222r

Explain what is happening to form the images (rather than just a single intensity as in the question above). What is the relationship between the two final images?

Extra credit 2: more fun with Fabry-Perot Interferometers and Optical Cavities
(an extension of problem 6)

A Fabry-Perot Interferometer can also store energy within it, as the light "leaking out" of the cavity through the mirrors must be less than the light that is reflected back into the FPI. Thus, the Fabry-Perot Interferometer is also an optical cavity—although most scientists consider it a poor cavity, in that it does not store that much light energy compared to cavities that were made for light storage.

How "good" a cavity is can be conveyed by its quality factor ("Q-factor"). The Q-factor is defined as

$$Q = 2\pi * \frac{Energy\ stored\ in\ cavity}{Energy\ lost\ per\ optical\ cycle}$$

a. Show that the Q-factor can also be found from the finesse and the optical frequency of the resonant light (ν_0), such that:
$$Q = Finesse * \frac{\nu_0}{FSR}$$
b. Using the Fabry-Perot cavity from the previous question, calculate the Q-factor. **Assume that the interferometer is 10 cm long and the light inside it has a wavelength of 550 nm (frequency of 545 THz). (Note: you will want FSR and ν_0 to have the same units; Q is unitless.)**
c. Suppose you want to make the Fabry-Perot cavity into a state-of-the-art optical cavity with a Q-factor of 1 million. What should the reflection coefficients, r, of the mirrors be? What fraction of light would they reflect (R = r^2)?